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URL: http://www.jjw3.com/r/byParts.pdf

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1 Integration by Parts

Used for integrals in the form $\int U(x)V(x)dx$, where $U'(x) \neq V(x)$ and $V'(x) \neq U(x)$.

Used also for $\int f(x)dx$, where $f(x)$ is one of the following:

- $\ln(x)$
- $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$, $\cot^{-1}(x)$

1.1 Definition

$\int UdV = UV - \int VdU$

The above definition may need to be applied more than once for a given integral (see example below).

1.2 Example

$\int x^2 e^x dx$

Let $U = x^2$, $dv = e^x dx$. dU in differential form is $dU = 2xdx$; $V = e^x$.
Using the definition of integration by parts:

$$\int x^2 e^x dx = x^2 e^x - \int e^x 2x dx$$

Need to integrate by parts again:
Let $U = 2x$, $dv = e^x dx$. dU in differential form is $dU = 2dx$; $V = e^x$.
Using the definition of integration by parts:

$$\int x^2 e^x dx = x^2 e^x - \int e^x 2x dx = x^2 e^x - \left(2xe^x - \int e^x 2dx\right)$$

The remaining integral can be evaluated using another method:

$\int x^2 e^x dx = x^2 e^x - \int e^x 2x dx = x^2 e^x - (2xe^x - 2e^x + C)$

Simplifying

$\int x^2 e^x dx = x^2 e^x - \int e^x 2x dx = x^2 e^x - 2xe^x + 2e^x + C$

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1.3 Table Method

$$\begin{array}{ccc} \underline{U} & & \underline{dV} \\ & & \\ x^2 & & e^x \\ & + & \\ 2x & & e^x \\ & - & \\ 2 & & e^x \\ & + & \\ 0 & & e^x \end{array}$$

Multiply along the diagonals and use sign along diagonals.

$$\int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x + C$$